OPERATIONS RESEARCH

Simulation

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Introduction

A model once constructed may be used to predict consequences of taking alternative actions. In particular, we could ‘experiment’ on the model by ‘trying’ alternative actions (or parameters) and compare their consequences. This ‘experimentation’ allows to answer ‘what if’ questions relating the effects of assumptions on the model response. Comparing the consequences by substituting various parameters into the model is referred to as *simulating* the model.

Simulation Defined

- *A simulation of a system or an organism is the operation of a model or simulator which is a representation of the system or organism. The model is amenable to manipulation which would be impossible, too expensive or unpractical to perform on the entity it portrays. The operation of the model can be studied and for it, properties concerning the behaviour of the actual system can be inferred.*

- This definition is broad enough to be applied equally to military war games, business games, economic models, etc. In this view simulation involves logical and mathematical constructs that can be manipulated on a digital computer using iterations or successive trials.

- *Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour (within the limits imposed by a criterion or set of criteria) for the operation of the system.* — Shannon

- *Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real-world system over extended periods of time.* — Naylor et al.

- *‘X simulated Y’ is true if and only if*
  a) *X and Y are formal systems,*
  b) *Y is taken to be the real system,*
  c) *X is taken to be an approximation to the real system, and*
  d) *The rules of validity in X are non-error-free, otherwise X will become the real system.*
- *Simulation is the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation.* — Churchman

Simulation – What it is/not

It is not a technique which should be applied in all cases. However, table below highlights what simulation is and what it is not.

It is	*It is not*
▪ a technique which uses computers. ▪ an approach for reproducing the processes by which events of chance and change are created in a computer. ▪ a procedure for testing and experimenting on models to answer what if ..., then so and so ... types of questions.	▪ an analytical technique which provides exact solution. ▪ A programming language but it could be programmed into a set of commands which can form a language to facilitate the programming of simulation.

Steps of Simulation Process

- **Identify the problem**: The simulation process is used to solve any problem only when the assumptions required for analytical models are not satisfied or there is no appropriate model developed for a system under study. For example, a queuing situation may be of interest but the arrival and/or service pattern do not meet the assumptions required to use queuing theory.

- **Identify the decision variables and decide the performance criterion (objective)**: In the context of an inventory control situation, the demand (consumption rate), lead time and safety stock are identified as decision variables. These variables shall be responsible to measure the performance of the system in terms of total inventory cost under the decision rule – when to order.

- **Construct a simulation model**: For developing a simulation model, an intimate understanding of the relationships among the elements of the system being studied is required. For this purpose the *influence diagram* (*drawn in a variety of different ways*) is useful because simulation models for each of these diagrams may be formulated until one seems better or more appropriate than the other. Even after one has been chosen, it may be modified again and again before a final version is acceptable.

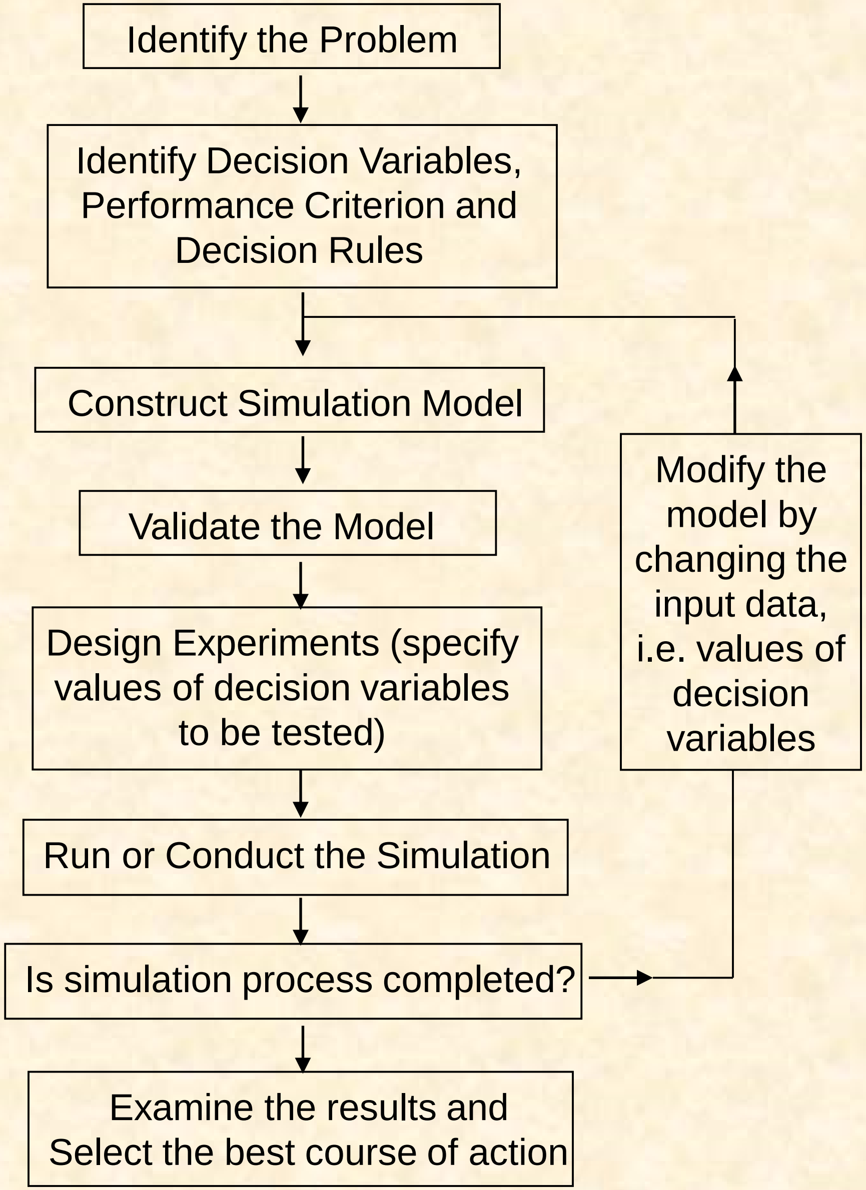
- **Testing and validating the model**: The validation process requires (i) determine whether the model is internally correct in a logical and programming sense called *internal validity* and (ii) determine whether it represents the system under study called *external validity*. The first step involves checking the equations and procedures in the model for accuracy, both in terms of mistakes (or errors) and in terms of properly representing the system under study. The verification of internal validity can be simplified if the model is developed in modules and each module is tested as it is developed.

  After verifying internal validity the model is tested by substituting historical values into the model and seeing if it replicates what happens in reality. If the model passes this test, extreme values of the input variables are entered and the model is checked for the expected output.

- **Designing of the experiment**: Experimental design refers to controlling the conditions of the study, such as the variables to include.

  It requires to determine factors considered fixed and variable in the model (ii) levels of the factors to use, (iii) what the resulting dependent measures are going to be, (iv) how many times the model will be replicated, and length of time of each replication, and so on. For example, in a queuing simulation we may consider arrival and service rates constant but vary the number of servers and the evaluate the customer waiting times. (dependent variable).

- **Run the simulation model**: Run the model on the computer to get the results in the form of operating characteristics.
- **Evaluate the results**: Examine the results of problem as well as their reliability and correctness. If the simulation process is complete, then select the best course of action (or alternative) otherwise make desired changes in model decision variables, parameters or design, and return to Step 3.



Steps of Simulation Process

Advantages and Disadvantages of Simulation

Advantages

- This approach is suitable to analyse large and complex real-life problems which cannot be solved by usual quantitative methods.
- It is useful for sensitivity analysis of complex systems. In other words, it allows the decision-maker to study the interactive system variables and the effect of changes in these variables on the system performance in order to determine the desired one.
- Simulation experiments are done with the model, not on the system itself. It also allows to include additional information during analysis that most quantitative models do not permit. In other words, simulation can be used to 'experiment' on a model of a real situation without incurring the costs of operating on the system.

- Simulation can be used as a pre-service test to try out new policies and decision rules for operating a system before running the risk of experimentation in the real system.
- The only ‘remaining tool’ when all other techniques become intractable or fail.

Disadvantages

- Sometimes simulation models are expensive and take a long time to develop. For example, a corporate planning model may take a long time to develop and prove expensive also.
- It is the trial and error approach that produces different solutions in repeated runs. This means it does not generate optimal solutions to problems.
- Each application of simulation is ad hoc to a great extent.
- The simulation model does not produce answers by itself. The user has to provide all the constraints for the solutions which he wants to examine.

Monte Carlo Simulation

The Monte Carlo simulation technique involved conducting repetitive experiments on the model of the system under study with some known probability distribution to draw random samples (observations) using random numbers. If a system cannot be described by a standard probability distribution, an empirical probability distribution can be constructed. The Monte Carlo simulation technique consists of following steps:

- Setting up a probability distribution for variables to be analysed.
- Building a cumulative probability distribution for each random variable.
- Generate random numbers and then assign an appropriate set of random numbers to represent value or range (interval) of values for each random variable.
- Conduct the simulation experiment using random sampling.
- Repeat Step 4 until the required number of simulation runs has been generated.
- Design and implement a course of action and maintain control.

Example: A book store wishes to carry a particular book in stock. Demand is not certain and there is a lead time of 2 days for stock replenishment. The probabilities of demand are given below:

Demand (units/day)	:	0	1	2	3	4
Probability	:	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Re 0.5 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day. The manager of the book store wishes to compare two options for his inventory decision.

A. Order 5 books when present inventory plus any outstanding order falls below 8 books.

B. Order 8 books when present inventory plus any outstanding order falls below 8 books.

Currently (beginning of 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and are expected to arrive next day. Carryout simulation run for 10 days to recommend an appropriate option. You may use random numbers in the sequences. Using first number for day one.

89, 34, 78, 63, 61, 81, 39, 16, 13, 73

Solution: Using the daily demand distribution, a probability distribution so obtained is as follows:

Daily Demand	*Probability*	*Cumulative Probability*	*Random Number Interval*
0	0.05	0.05	00–04
1	0.10	0.15	05–14
2	0.30	0.45	15–44
3	0.45	0.90	45–89
4	0.10	1.00	90–99

Given that stock in hand is of 8 books and stock on order is 5 books (expected next day).

Option A

Random Number	*Demand Daily*	*Closing Stock in Hand*	*Receipt*	*Opening Stock in Hand*	*Stock on Order*	*Order Quantity*	*Closing Stock*
89	3	8	–	8 – 3 = 5	6	–	6
34	2	5	6	6 + 5 – 2 = 9	–	–	–
78	3	9	–	9 – 3 = 6	–	5	5
63	3	6	–	6 – 3 = 3	5	–	5
61	3	3	–	3 – 3 = 0	5	5	10
81	3	0	5	5 – 3 = 2	5	5	10
39	2	2	–	2 – 2 = 0	10	–	10
16	2	0	5	5 – 2 = 3	5	–	5
13	1	3	5	5 + 3 – 1 = 7	0	5	5
73	3	7	–	7 – 3 = 4	5	–	5

Since 5 books have been ordered four times shown in previous table, total ordering cost is Rs (4 × 10) = Rs 40.

Option B

Random Number	*Demand Daily*	*Closing Stock in Hand*	*Receipt*	*Opening Stock in Hand*	*Stock on Order*	*Order Quantity*	*Closing Stock*
89	3	8	–	8 – 3 = 5	6	–	6
34	2	5	6	6 + 5 – 2 = 9	–	–	–
78	3	9	–	9 – 3 = 6	–	8	8
63	3	6	–	6 – 3 = 3	8	–	8
61	3	3	–	3 – 3 = 0	8	–	8
81	3	0	8	8 + 0 – 3 = 5	–	8	8
39	2	5	–	5 – 2 = 3	8	–	8
16	2	3	–	3 – 2 = 1	8	–	8
13	1	1	8	8 + 1 – 1 = 8	–	–	–
73	3	8	–	8 – 3 = 5	–	8	8

Since 8 books have been ordered four times (when the inventory of books at the beginning of the day plus orders outstanding is less than 8), therefore, total ordering cost is Rs (3 × 10) = Rs 30.

Closing stock of 10 days is of 45 (= 5 + 9 + 6 + 3 + 5 + 3 + 1 + 8 + 5) books. Therefore, holding cost, Re 0.5 per book per day is Rs (45 × 0.5) = Rs 22.50.

Total cost for 10 days = Ordering cost + Holding cost = Rs 52.50. Since option B has lower total cost than option A, manager should choose option B.

Example: The manager of a warehouse is interested in designing an inventory control system for one of the products in stock. The demand for the product comes from numerous retail outlets and orders arrive on a weekly basis. The warehouse receives its stock from the factory but the lead time is not constant. The manager wants to determine the best time to release orders to the factory so that stockouts are minimized, yet inventory holding costs are at acceptable levels. Any order from retailers not supplied on a given day constitute lost demand. Based on a sampling study, the following data are available.

Demand per Week (in thousand)	*Probability*	*Lead Time*	*Probability*
0	0.20	2	0.30
1	0.40	3	0.40
2	0.30	4	0.30
3	0.10		

The manager of the warehouse has determined the following cost parameters: Ordering cost (C_0) per order equals Rs 50, carrying cost (C_h) equals Rs 2 per thousand units per week, and shortage cost (C_s) equals Rs 10 per thousand units.

The objective of inventory analysis is to determine the optimal size of an order and the best time to place an order.

Policy: Whenever the inventory level becomes less than or equal to 2,000 units (reorder level), an order equal to the difference between current inventory balance and the specified maximum replenishment level is equal to 4,000 units is placed.

Simulate the policy for a week's period assuming that the (i) beginning inventory is 3,000 units, (ii) no back orders are permitted, (iii) each order is placed at the beginning of the week as soon as inventory level is less than or equal to the reorder level, and (iv) replenishment orders are received at the beginning of the week.

Solution: Using weekly demand and lead time distributions, assign an appropriate set of random numbers to represent value (range) of variables as shown below:

Probabilities and Random Number Interval for Weekly Demand

Weekly Demand (in thousand)	*Probability*	*Cumulative Probability*	*Random Number Interval*
0	0.20	0.20	00–19
1	0.40	0.60	20–59
2	0.30	0.90	60–89
3	0.10	1.00	90–99

Probabilities and Random Number Interval for Lead Time

Lead Time (weeks)	*Probability*	*Cumulative Probability*	*Random Number Interval*
2	0.30	0.30	00–29
3	0.40	0.70	30–69
4	0.30	1.00	70–99

The simulation process begins with an inventory level of 3,000 units. The following four steps occur in the simulation process.

- Begin each simulation week by checking whether any order has just arrived. If it has, increase the beginning (current) stock (inventory) by the quantity received.
- Generate a weekly demand from the demand probability distribution by selection of a random number, recorded in column 4. The demand simulated is recorded in column 5. The random number 31 generates a demand of 1,000 units when it is subtracted from the initial inventory level value of 3,000 units, yields an ending inventory of 2,000 units at the end of the first week.
- Compute the ending inventory every week and record it in column 7.

  Ending inventory = Beginning inventory–Demand = 3,000 – 1,000 = 2,000

  If on hand inventory is not sufficient to meet the week's demand, then record the number of units short in column 6.

- Determine whether the week's ending inventory has reached the reorder level. If it has, and if there is no outstanding (back orders), then place an order. Since ending inventory of 2,000 units is equal to the reorder level, an order for 4,000 – 2,000 = 2,000 units is placed.
- The lead time for the new order is simulated by first choosing a random number and recording it in column 8. Finally, this random number is converted into a lead time (column 9) by using the lead time distribution in above table. The random number 29 corresponds to a lead time of 2 weeks. With 2,000 units to be held (carried) in stock, the holding cost of Rs 4 is paid and since there were no shortages, there is no shortage cost. Summing these cost yields a total inventory cost (column 10) for week one of Rs 54.

Repeat simulation experiment for 10 weeks.

Inventory Simulation Experiments

Maximum Inventory Level = 4,000 units *Reorder Level = 2,000 units*

Week	*Order Receipt*	*Beginning Inventory*	*Random Number*	*Demand*	*Ending Inventory*	*Quantity Ordered*	*Random Number*	*Lead Time*	*Total Cost (TC)* C_0	+ C_h	+ C_s	=TC (Rs)
1	0	3,000	31	1,000	2,000	2,000	29	2	50	4	–	= 54
2	0	2,000	70	2,000	0	0	–	--	--	--	--	--
3	0	0	53	1,000	(– 1,000)	0	–	–	0	0	10	= 10
4	2,000	2,000	86	2,000	0	4,000	83	4	50	–	–	= 50
5	0	0	32	1,000	(– 1,000)	0					10	= 10
6	0	0	78	2,000	(– 2,000)	0					20	= 20
7	0	0	26	1,000	(– 1,000)	0					10	= 10
8	0	0	64	2,000	(– 2,000)	0					20	= 20
9	4,000	4,000	45	1,000	3,000	0				6	–	= 06
10	0	3,000	12	0	3,000	0				6	–	= 06
			Total		1,000				100	16	70	

$$\text{Average ending inventory} = \frac{1{,}000 \text{ total unit}}{10 \text{ weeks}} = 100 \text{ units per week.}$$

$$\text{Average number of orders placed} = \frac{2 \text{ orders}}{10 \text{ weeks}} = 0.2 \text{ order per week.}$$

$$\text{Average number of lost sales} = \frac{7{,}000}{1{,}000} = 7 \text{ units per week.}$$

Total average inventory cost = Ordering cost + Holding cost + Shortage cost

= (Cost of placing one order) × (Number of orders placed per week) + (Cost of holding one unit for one week) × (Average ending inventory) + (Cost per lost sale) × (Average number of lost sales per week)

$$= \frac{100}{10} + \frac{16}{10} + \frac{70}{10} = 10 + 1.6 + 7 = \text{Rs } 18.6$$

Simulation of Queuing Problems

Example: A dentist schedules all his patients for 30-minute appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and time actually needed to complete the work:

Category of Service	*Time Required (minutes)*	*Probability of Category*
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Checkup	15	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 a.m. Use the following random numbers for handling the above problem: 40 82 11 34 25 66 17 79

Solution The cumulative probability distribution and random number interval for service time are shown below:

Category of Service	*Service Time Required (minutes)*	*Probability*	*Cumulative Probability*	*Random Number Interval*
Filling	45	0.40	0.40	00–39
Crown	60	0.15	0.55	40–54
Cleaning	15	0.15	0.70	55–69
Extraction	45	0.10	0.80	70–79
Checkup	15	0.20	1.00	80–99

The various parameters of a queuing system such as arrival pattern of customers, service time, waiting time in the context of the given problem are shown below:

Arrival Pattern and Nature of Service

Patient Number	*Scheduled Arrival*	*Random Number*	*Category of Service*	*Service Time (minutes)*
1	8.00	40	Crown	60
2	8.30	82	Checkup	15
3	9.00	11	Filling	45
4	9.30	34	Filling	45
5	10.00	25	Filling	45
6	10.30	66	Cleaning	15
7	11.00	17	Filling	45
8	11.30	79	Extraction	45

Computation of Arrivals, Departures and Waiting of Patients

Time	*Event* (*Patient Number*)	*Patient Number* (*Time to Exit*)	*Waiting* (*Patient Number*)
8.00	1 arrive	1 (60)	–
8.30	2 arrive	1 (30)	2
9.00	1 departs; 3 arrive	2 (15)	3
9.15	2 depart	3 (45)	–
9.30	4 arrive	3 (30)	4
10.00	3 depart; 5 arrive	4 (45)	5
10.30	6 arrive	4 (15)	5, 6
10.45	4 depart	5 (45)	6
11.00	7 arrive	5 (30)	6, 7
11.30	5 depart; 8 arrive	6 (15)	7, 8
11.45	6 depart	7 (45)	8
12.00	End	7 (30)	8

Average Waiting Time for Dentists

Patient	*Arrival Time*	*Service Starts at*	*Waiting Time (minutes)*
1	8.00	8.00	0
2	8.30	9.00	30
3	9.00	9.15	15
4	9.30	10.00	30
5	10.00	10.45	45
6	10.30	11.30	60
7	11.00	11.45	45
8	11.30	12.30	60
			280

The average waiting time = 280/8 = 35 minutes.

Example A firm has a single channel service station with the following arrival and service time probability distributions:

Interarrival Time (minutes)	*Probability*	*Service Time (minutes)*	*Probability*
10	0.10	5	0.08
15	0.25	10	0.14
20	0.30	15	0.18
25	0.25	20	0.24
30	0.10	25	0.22
		30	0.14

The customer's arrival at the service station is a random phenomenon and the time between the arrivals varies from 10 minutes to 30 minutes. The service time varies from 5 minutes to 30 minutes. The queuing process begins at 10 a.m. and proceeds for nearly 8 hours. An arrival goes to the service facility immediately, if it is free. Otherwise it will wait in a queue. The queue discipline is first-come first-served.

If the attendant's wages are Rs 10 per hour and the customer's waiting time costs Rs 15 per hour, then would it be an economical proposition to engage a second attendant? Answer using Monte Carlo simulation technique.

Solution The cumulative probability distributions and random number interval both for interarrival time and service time are as follows:

Inter-arrival Time

Interarrival Time (minutes)	*Probability*	*Cumulative Probability*	*Random Number Interval*
10	0.10	0.10	00–09
15	0.25	0.35	10–34
20	0.30	0.65	35–64
25	0.25	0.90	65–89
30	0.10	1.00	90–99

Interarrival Time (minutes)	*Probability*	*Cumulative Probability*	*Random Number Interval*
5	0.08	0.08	00–07
10	0.14	0.22	08–21
15	0.18	0.40	22–39
20	0.24	0.64	40–63
25	0.22	0.86	64–85
30	0.14	1.00	86–99

Single Server Queuing Simulation for 15 Arrivals

Arrival Number (1)	*Random Number* (2)	*Arrival Interval* (3)	*Arrival Time* (4)	*Service Time* (5)	*Waiting Time* (6)	*Random Number* (7)	*Service Time* (8)	*Exit Time* (9)	*Time in System* (10)= (6) + (8)
1	20	15	15	15	0	26	15	30	15
2	73	25	40	40	0	43	20	60	20
3	30	15	55	60	5	98	30	90	35
4	99	30	85	90	5	87	30	120	35
5	66	25	110	120	10	58	20	140	30
6	83	25	135	140	5	90	30	170	35
7	32	15	150	170	20	84	25	195	45
8	75	25	175	195	20	60	20	215	40
9	04	10	185	215	30	08	10	225	40
10	15	15	200	225	25	50	20	245	45
11	29	15	215	245	30	37	15	260	45
12	62	20	235	260	25	42	20	280	45
13	37	20	255	280	25	28	15	295	40
14	68	25	280	295	15	84	25	320	40
15	94	30	310	320	10	65	25	345	35

From 15 samples waiting time 225 minutes and time spent 545 minutes by the customer in the system, then;

Average waiting time = 225/15 = 15 minutes.

Average service time = 545/15 = 36.33 minutes

Thus, the average cost of waiting and service is given by

Cost of waiting	$= 15 \times (15/60) =$ Rs 3.75 per hour
Cost of service	$= 10 \times (36.33/60) =$ Rs 6.05 per hour

Since average cost of service per hour is more than the average cost of waiting per hour, it would not be an economical proposition to engage a second attendant.

Simulation of Investment Problems

Example The Investment Corporation wants to study the investment projects based on three factors: market demand in units; price per unit minus cost per unit and investment required. These factors are felt to be independent of each other. In analysing a new consumer product, the Corporation estimates the following probability distributions:

Annual Demand		*Price minus Cost per Unit*		*Investment Required*	
Units	*Probability*	*Rs*	*Probability*	*Rs*	*Probability*
20,000	0.05	3.00	0.10	17,50,000	0.25
25,000	0.10	5.00	0.20	20,00,000	0.50
30,000	0.20	7.00	0.40	25,00,000	0.25
35,000	0.30	9.00	0.20		
40,000	0.20	10.00	0.10		
45,000	0.10				
50,000	0.05				

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial taking these three factors into account. What is the most likely return?

Solution The return per annum can be computed by the following expression

$$\text{Return (R)} = \frac{(\text{Price} - \text{Cost}) \times \text{Number of units demanded}}{\text{Investment}}$$

Developing a cumulative probability distribution corresponding to each of the three factors, an appropriate set of random numbers is assigned to represent each of the three factors as shown below:

Annual Demand	*Probability*	*Cumulative Probability*	*Random Number*
20,000	0.05	0.05	00–04
25,000	0.10	0.15	05–14
30,000	0.20	0.35	15–34
35,000	0.30	0.65	35–64
40,000	0.20	0.85	65–84
45,000	0.10	0.95	85–94
50,000	0.05	1.00	95–99

Price minus Cast per Unit	*Probability*	*Cumulative Probability*	*Random Number*
3.00	0.10	0.10	00–09
5.00	0.20	0.30	10–19
7.00	0.40	0.70	20–69
9.00	0.20	0.90	70–89
10.00	0.10	1.00	90–99

Investment Required	*Probability*	*Cumulative Probability*	*Random Number*
17,50,000	0.25	0.25	00–24
20,00,000	0.50	0.75	25–74
25,00,000	0.25	1.00	75–99

The simulation worksheet is prepared for 10 trials. The simulated return (R) is also calculated by using the formula for R as stated before. The results of simulation are shown below:

Trials	*Random Number for Demand*	*Simulated Demand ('000)*	*Random Number for Profit (Price – Cost) per Unit*	*Simulated Profit*	*Random Number for Investment*	*Simulated Investment ('000)*	*Simulated return (%): Demand × Profit per Unit / Investment X 100*
1	28	30	19	5.00	18	1,750	8.57
2	57	35	07	3.00	61	2,000	5.25
3	60	35	90	10.00	16	1,750	20.00
4	17	30	02	3.00	71	2,000	4.50
5	64	35	57	7.00	43	2,000	12.25
6	20	30	28	5.00	68	2,000	7.50
7	27	30	29	5.00	47	2,000	7.50
8	58	35	83	9.00	24	1,750	18.00
9	61	35	58	7.00	19	1,750	14.00
10	30	30	41	7.00	97	2,500	8.40

As shown above, the highest likely return is 20 per cent which corresponds to annual demand of 35,000 units yielding a profit of Rs 10 per unit and investment required is Rs 17,50,000.

Simulation of Maintenance Problems

Example A plant has a large number of similar machines. The machine breakdowns or failures are random and independent.

The shift in-charge of the plant collected the data about the various machines breakdown times and the repair time required on hourly basis, and the record for the past 100 observations as shown below was:

Time Between Recorded Machine Breakdowns (hours)	*Probability*	*Repair Time Required (hours)*	*Probability*
0.5	0.05	1	0.28
1	0.06	2	0.52
1.5	0.16	3	0.20
2	0.33		
2.5	0.21		
3	0.19		

For each hour that one machine is down due to being or waiting to be repaired, the plant loses Rs 70 by way of lost production. A repairman is paid at Rs 20 per hour.

a) Simulate this maintenance system for 15 breakdowns.
b) How many repairmen should the plant hire for repair work.

Solution The random numbers coding for the hourly breakdowns and the repair times are shown in Tables.

Random Number Coding for Breakdowns

Time Between Breakdowns (hours)	*Probability*	*Cumulative Probability*	*Random Number Range*
1	0.28	0.28	00–27
2	0.52	0.80	28–79
3	0.20	1.00	80–99

Repair Time Required (hours)	*Probability*	*Cumulative Probability*	*Random Number Range*
1	0.28	0.28	00–27
2	0.52	0.80	28–79
3	0.20	1.00	80–99

In simulation sheet, it is assumed that the first day begins at midnight (00.00 hours) and also the repairman begins work at 00.00 hours. The first breakdown occurred at 2.30 a.m and the second occurred after 3 hours at clock time of 5.30 a.m.

Simulation Worksheet

Breakdown Number	Random Number for Break-downs	Time Between Break-downs	Time of Break-down	Repair Work Begins at	Random Number for Repair Time	Repair Time Required	Repair Work Ends at	Total Idle Time (hours)	Waiting Time (hours)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	61	2.5	02.30	02.30	87	3	05.30	3.00	–
2	85	3	05.30	05.30	39	2	07.30	2.00	–
3	16	1.5	07.00	07.30	28	2	09.30	2.30	0.30
4	46	2	09.00	09.30	97	3	12.30	3.30	0.30
5	88	3	12.00	12.30	69	2	14.30	2.30	0.30
6	08	1	13.00	14.30	87	3	17.30	4.30	1.30
7	82	3	16.00	17.30	52	2	19.30	3.30	1.30
8	56	2	18.00	19.30	52	2	21.30	3.30	1.30
9	22	1.5	19.30	21.30	15	1	22.30	3.00	2.00
10	49	2	21.30	22.30	85	3	01.30	4.00	1.00
11	44	2	23.30	01.30	41	2	03..30	4.00	2.00
12	33	2	01.30	03.30	82	3	06.30	5.00	2.00
13	77	2.5	04.00	06.30	98	3	09.30	5.30	2.30
14	87	3	07.00	09.30	99	3	12.30	5.30	2.30
15	54	2	09.00	12.30	23	2	14.30	5.30	3.30
				38.30		36		57.30	21.30

Total current maintenance cost = Idle time cost + Repairman's wage

+ (Repair time + Waiting time) × Hourly rate

+ Total hours × Hourly wages

= 57.30 × 70 + 38.30 × 20 = Rs 4,777

Maintenance Cost with Additional Repairman: If the plant hires two more repairmen, then no machine will wait for its repair. Thus, total idle time would be only the repairing time of 36.00 hours. Therefore,

Total cost = 36 × 70 + (38.30 × 2) × 20 = Rs 4,052

This shows that hiring more than two repairmen would only increase the total maintenance cost. Hence, the plant may hire one additional repairman.